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Research supported by this grant focussed on the regularity of the value function for multi-dimensional singular stochastic control problems. In addition, a multi-dimensional finite-fuel problem has been studied, and known results for one-dimensional problems have been extended to higher dimensions.

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FINAL REPORT
SINGULAR AND BANG-BANG STOCHASTIC CONTROL
AFOSR GRANT NUMBER 89-0075
October 1, 1988 - September 30, 1990

Research supported by this grant focussed on the regularity of the value function for multi-dimensional singular stochastic control problems. In addition, a multi-dimensional finite-fuel problem has been studied, and known results for one-dimensional problems have been extended to higher dimensions. Each of these topics is discussed in more detail below.

In singular stochastic control, the Hamilton-Jacobi-Bellman conditions used to characterize the value function take the form of a variational inequality whose solution involves the determination of a free boundary. The proof that the value function satisfies this variational inequality is generally quite technical, involving the regularity of the value function itself. We have long suspected that under very general conditions, the value function is twice continuously differentiable, and thus the Bellman variational inequality can be interpreted in the classical sense. For one-dimensional problems, this C^2 property of the value function can often be used to determine it explicitly; in higher dimensions it is instrumental in the construction of the optimal control process. We also believe that this property will be useful in the design of numerical algorithms for solving these control problems.

Under the present grant, we have obtained C^2 regularity for the value function of a two-dimensional, infinite-horizon, discounted problem in which control could be exercised in any direction [1], and we have obtained a similar result for an n-dimensional, infinite-horizon, discounted problem in which control can cause displacement in one direction only [2] and also for the finite-horizon version of the latter problem [3]. In all these models, the optimal control process has been shown to exist and has been characterized in terms of the value function.

A second singular stochastic control problem under study is the finite-fuel control of a multi-dimensional Brownian motion. Originally posed by Beneš, Shepp and Witsenhausen, who solved the one-dimensional version, this problem involves pushing a Brownian motion toward the origin so as to minimize some objective, such as the integrated, discounted, square of the norm of the controlled process, over an infinite horizon. However, there is a prespecified upper bound on the total amount of pushing which can be exercised, i.e., there is a finite-fuel constraint. Although the multi-dimensional problem is similar in spirit to the one solved by Beneš et al, the actual verification of the form of the optimal solution turns out to be surprisingly complex. At this point, the problem has been solved and we are in the process of writing up the results, which will appear in [4]. The primary researcher on this problem is David Bridge, a Ph.D. candidate in the Carnegie Mellon Department of Mathematics.

[1] S. Shreve., & Soner, M., Regularity of the value function for a two-dimensional singular stochastic control problem, **SIAM J. Control and Optimization 27**, (1989), 876-907.

[2] Shreve, S., & Soner, M., A free boundary problem related to singular stochastic control, **Proc. 1989 Imperial College Workshop on Applied Stochastic Analysis**, M. H. A. Davis and J. M. C. Clark, ed., Gordon & Breach, London, to appear.

[3] Shreve, S., & Soner, M., A free boundary problem related to singular stochastic control: the parabolic case, **Comm. Partial Differential Equations**, to appear.

[4] Bridge, D., Finite-Fuel Singular Stochastic Control of an n-Dimensional Infinite-Horizon Discounted Problem, Ph.D. Dissertation, Dept. of Mathematics, Carnegie Mellon Univ